

The use of cortical field potentials rather than the details of spike trains as the basis for cognitive information processing is proposed. This results in a space of cognitive elements with natural metrics. Sets of spike trains may also be considered to be points in a multidimensional metric space. The closeness of sets of spike trains in such a space implies the closeness of points in the resulting function space of potential distributions.

1. Introduction

Nearly all theories of information processing focus attention on dynamical patterns of action potentials [1] including studies which involve correlational analyses [2]. In many of these studies authors choose to regard spikes as temporally discrete events and consider their rates or temporal relationships to be significant. (In reality action potentials are continuous, at least within the framework of classical physics.) The usual approach leads to difficulties in the construction of metrics for cortical activity because metrics for sequences (of time points) give large distances if minor differences occur between spike trains. The latter seems to imply that spike trains themselves are not a useful description per se in the description of cortical activity.

2. Potential distributions and information processing

Rather than focus on spikes and in particular their times of occurrence as a point process, we concentrate on the *field potentials* they are associated with, or generate, in a region $M \subset R^3$ of cerebral cortex or other brain structure. It is also convenient to restrict attention to a finite time interval $T = [t_1, t_2] \subset R^+$, say.

Firstly we consider the actual electrical potential distribution $V(\mathbf{x}, t)$, $\mathbf{x} \in M$, $t \in T$, throughout the region. For convenience only extracellular points of M will be considered, but we will continue to use the same symbol M for the spatial region. The exact potential V cannot be measured exactly but is able to be estimated by an approximate *field potential* [3] over a region $A_{\mathbf{x}}$ surrounding the space point \mathbf{x} , (and probably also a small time interval surrounding t)

$$V_E(\mathbf{x}, t) = \frac{1}{|A_{\mathbf{x}}|} \int \int \int_{A_{\mathbf{x}}} V(\mathbf{x}', t) w(\mathbf{x}', t) d\mathbf{x}',$$

where $|\cdot|$ denotes volume. The region $A_{\mathbf{x}}$ reflects the size of a recording electrode and the weight function $w(\mathbf{x}, t)$ reflects its electrical properties.

Now the set of all possible potential distributions $V(\mathbf{x}, t)$ on $M \times T$, which we denote by $C_V(M \times T)$, is a set of bounded continuous functions and a subset of the space $C(M \times T)$ of all bounded continuous functions on that product space. For such a space there are suitable metrics. Let V_1 and V_2 be two points in C_V (i.e., potential distributions). Then one metric (distance function) is provided by the uniform or sup norm,

$$d_1(V_1, V_2) = \sup_{M \times T} |V_1(\mathbf{x}, t) - V_2(\mathbf{x}, t)|.$$

Alternatively and perhaps more satisfactorily we may consider C_V as a subset of the space of square integrable functions on $M \times T$ in which case we may use the metric

$$d_2(V_1, V_2) = \left[\int_M \int_T (V_1(\mathbf{x}, t) - V_2(\mathbf{x}, t))^2 dt d\mathbf{x} \right]^{\frac{1}{2}}.$$

However, if one is only interested in comparing potential distributions at a given time t and therefore considered only to be functions of \mathbf{x} , then the following corresponding metrics will be useful:

$$D_1(V_1, V_2) = \sup_M |V_1(\mathbf{x}, t) - V_2(\mathbf{x}, t)|$$

and

$$D_2(V_1, V_2) = \left[\int_M (V_1(\mathbf{x}, t) - V_2(\mathbf{x}, t))^2 d\mathbf{x} \right]^{\frac{1}{2}}.$$

3. Applications

Consider a stimulus S_0 , of extrinsic or intrinsic origin, or a combination of both. With this stimulus will be associated a set of spike trains, constituting a point in a metric space - see below. There will also be an associated potential distribution which we assume is in the region $M \times T$. However it is very unlikely that either the set of spike trains or the potential distribution are uniquely determined by S_0 as the response to the same stimulus is never exactly the same at different presentations. Thus there will be an average potential distribution associated with S_0 which we denote by $\overline{V}_0(\mathbf{x}, t)$, $\mathbf{x} \in M, t \in T$. If now a stimulus S occurs, it will be identified with S_0 if the potential distribution elicited, V , satisfies

$$d_1(V, \overline{V}_0) < \epsilon_1,$$

or

$$d_2(V, \overline{V}_0) < \epsilon_2,$$

where the positive constants ϵ_1 and ϵ_2 are measures of the discriminatory ability of cognitive processes.

Spike Trains

Suppose there are n neurons in the region M and in response to the stimulus S let the k -th of these have spikes at times $t_{k,1}, t_{k,2}, \dots, t_{k,n_k}$ where all these time points are in T . Let, for $t \in T$, $N_k(t)$ be the number of spikes of neuron k in $(t_1, t]$. Then $\{N_k(t), t \in [t_1, t_2]\}$ is, for each k , a right-continuous function on T and is an element of the space $D(T)$ of functions which are at each point in T right-continuous and with left-hand limits (*cadlag*). $D(T)$ is a metric space with the uniform norm. Thus we may consider the whole set of action potentials in M in T as a point in the space $D^n(T)$. Let y_1 and y_2 be two points in $D^n(T)$. Then the distance between these two sets of action potentials is

$$\rho(y_1, y_2) = \sum_{k=1}^n \rho_k$$

where ρ_k is the distance between the responses in the k -th spike train (supremum on T for each component). We claim that if stimuli S_1 and S_2 lead to sets of spike trains y_1 and y_2 , then there will be a δ such that

$$\rho(y_1, y_2) < \delta \Rightarrow d_1(V_1, V_2) < \epsilon_1.$$

That is distinguishable stimuli lead to distinguishable sets of spike trains which are components of distinguishable potential distributions. Differences in spike train details are expected to be smoothed out so that minor differences are not relevant for cognitive information processing.

Another possibility is that the space C_V is partitioned into a set of disjoint subsets C_ϵ and that when a potential distribution falls within C_ϵ , the corresponding cognitive element pertains.

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